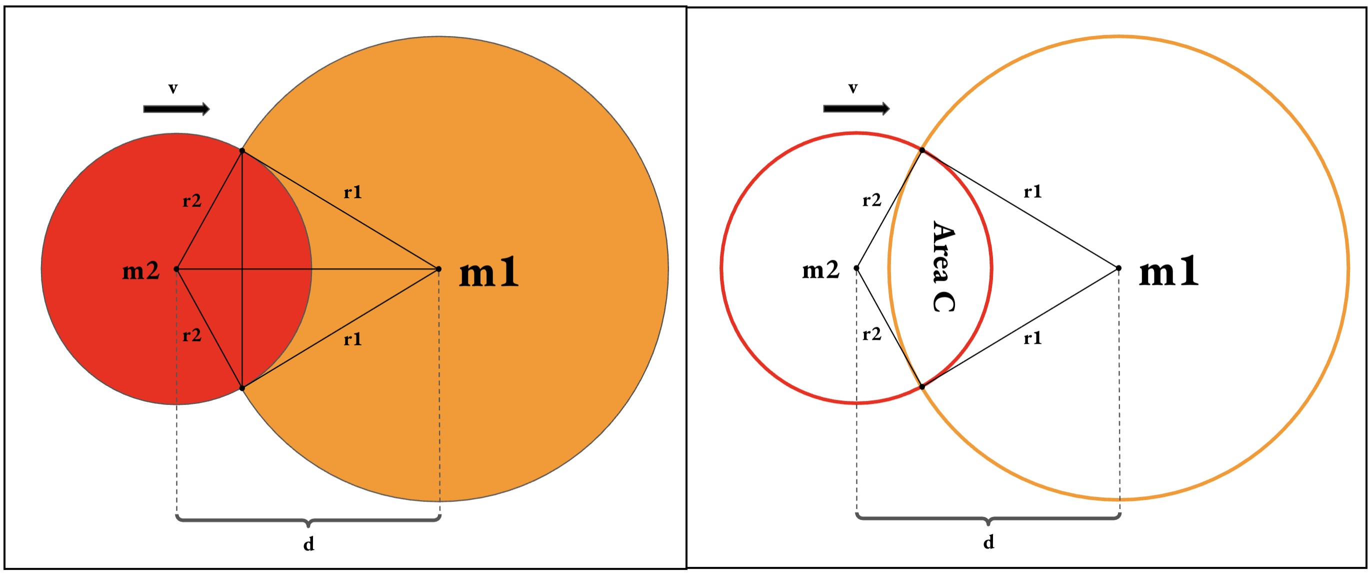
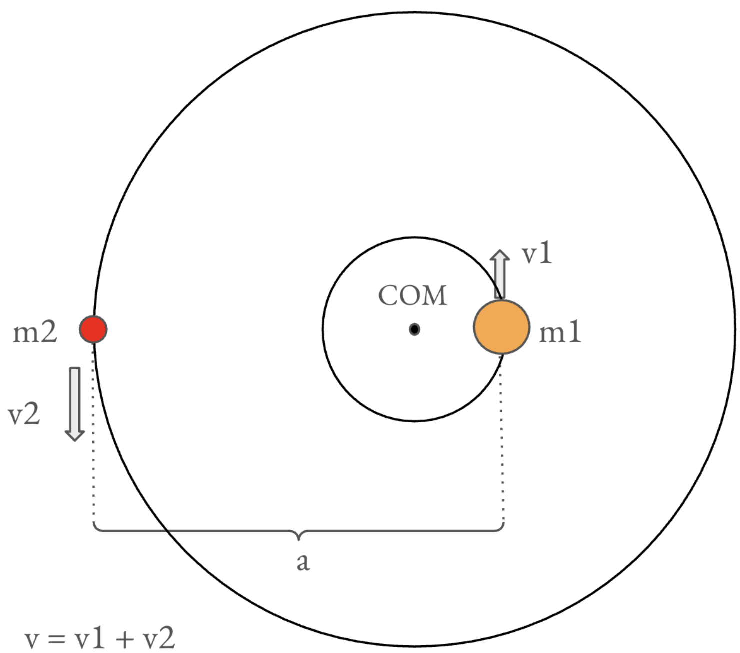
(1) Major Assumptions:

- Uniform Luminosity Distribution (No limb darkening, luminosity/projected area = const)
- Tangential Velocity is constant
- Circular Orbit (e = 0)
- Orbital Inclination is 90 deg
- Distance to System >> Orbital Radii
- Celestial bodies m1 and m2 is perfectly spherical with projected areas being perfect circles





(2) **To determine $L_E(t)$, the luminosity of the system during the eclipse as a function of time, it requires $A_C(t)$, the area eclipsed as a function of time.**

The area bounded by a circle-circle intersection is:

$$A = r_1^{\,2} arccos\left(\frac{d^2+r_1^{\,2}-r_2^{\,2}}{2dr_1}\right) + r_2^{\,2} arccos\left(\frac{d^2+r_2^{\,2}-r_1^{\,2}}{2dr_2}\right) - \frac{1}{2}\sqrt{\left(-d+r_2+r_1\right)\left(d+r_2-r_1\right)\left(d-r_2+r_1\right)\left(d+r_2+r_1\right)}$$

$$A_C = r_1^{\,2} arccos\left(\frac{d^2+r_1^{\,2}-r_2^{\,2}}{2dr_1}\right) + r_2^{\,2} arccos\left(\frac{d^2+r_2^{\,2}-r_1^{\,2}}{2dr_2}\right) - \frac{1}{2}\sqrt{\left(d^2-\left(r_2-r_1\right)^2\right)\left(\left(r_1+r_2\right)^2-d^2\right)}$$

Relative velocity between m_1 and m_2 is $v = \sqrt{\frac{G\left(m_1+m_2\right)}{a}}$, assuming a completely circular orbit (e = 0)

$d(t) = \left(r_1 + r_2\right) - vt$, where eclipse begins at t = 0

$$A_C(t) = r_1^{\,2} arccos\left(\frac{d(t)^2+r_1^{\,2}-r_2^{\,2}}{2d(t)r_1}\right) + r_2^{\,2} arccos\left(\frac{d(t)^2+r_2^{\,2}-r_1^{\,2}}{2d(t)r_2}\right) - \frac{1}{2}\sqrt{\left(d(t)^2-\left(r_2-r_1\right)^2\right)\left(\left(r_1+r_2\right)^2-d(t)^2\right)}$$

If m_2 eclipsing m_1

$$L_{Eclipse} = \frac{A_{Projected2}}{A_2}L_2 + \frac{A_{Projected1}}{A_1}L_1 \Rightarrow L_2 + \frac{\pi r_1^{\,2}-A_C(t)}{\pi r_1^{\,2}}L_1$$

The eclipse transit is defined as

$$T_{Duration} = \frac{P}{\pi} arcsin\left(\frac{r_1+r_2}{a}\right)$$

which accounts for the change in tangential velocity during the eclipse

However for $a \gg r_1 + r_2$, the change in tangential velocity during the eclipse is negligible

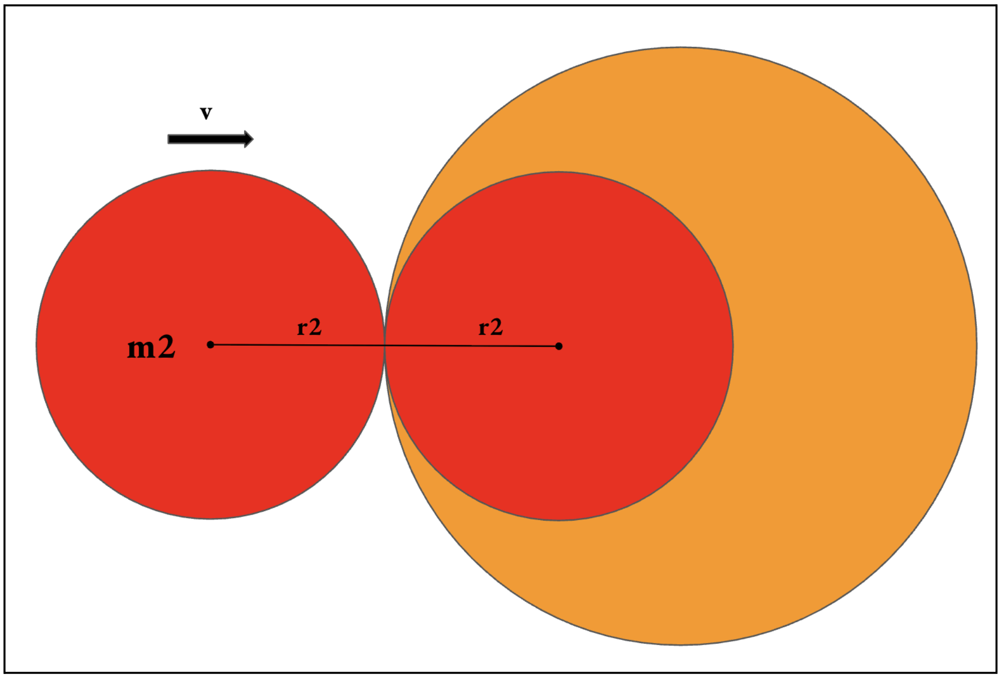
$$T_{Duration} = \frac{P}{\pi} arcsin\left(\frac{r_1+r_2}{a}\right) \approx T_0 = \frac{2r_1+2r_2}{v}$$

For simplicity, assume that at the time intervals where $\frac{d}{dt}\left[A_C(t)\right] \neq 0$,

$$Tangential\,Velocity = v = \sqrt{\frac{G\left(m_1+m_2\right)}{a}}, \qquad \frac{dv}{dt} = 0$$

Such that the time for m_2 to cross fully into the disk of m_2 is $\frac{2r_2}{v}$.

The same is true for when m_2 exits m_1's disk



Because we assume that when v is constant when $\frac{d}{dt}\left[A_C(t)\right] \neq 0$ and $T_{Duration} = \frac{P}{\pi} arcsin\left(\frac{r_1+r_2}{a}\right) \approx \frac{2r_1+2r_2}{v}$ accounts for the change in v (tangential velocity), the difference between the estimations is

$$T_E = T_{Duration} - T_0$$

Where $T_{Duration} > T_0$ is always true

(3) Combining $L_{Eclipse}$, $A_C(t)$, $d(t)$, v, $T_{Duration}$, T_E, and the assumptions made in (1),

we are able to create $L(t)$,

the luminosity of the system as a function of time during the eclipse.

$$L(t) = \left\{ \begin{array}{ll} \left(1 - \frac{A_C(t)}{\pi {r_1}^2}\right)L_1 + L_2, & 0 \le t \le \frac{2r_2}{v} \\ \left(1 - \frac{r^2}{R^2}\right)L_1 + L_2, & \frac{2r_2}{v} \le t \le \frac{2r_1}{v} + T_E \\ \left(1 - \frac{A_C\left(T_{Duration} - t\right)}{\pi {r_1}^2}\right)L_1 + L_2, & \frac{2r_1}{v} + T_E \le t \le T_{Duration} \end{array} \right\}$$

Where:

- $A_C(t) = {r_1}^2 arccos\left(\frac{d(t)^2 + {r_1}^2 - {r_2}^2}{2d(t)r_1}\right) + {r_2}^2 arccos\left(\frac{d(t)^2 + {r_2}^2 - {r_1}^2}{2d(t)r_2}\right) - \frac{1}{2}\sqrt{\left(d(t)^2 - \left(r_2 - r_1\right)^2\right)\left(\left(r_1 + r_2\right)^2 - d(t)^2\right)}$
- $d(t) = \left(r_1 + r_2\right) - vt$, where eclipse begins at t = 0
- $T_{Duration} = \frac{P}{\pi} arcsin\left(\frac{r_1 + r_2}{a}\right)$, $T_0 = \frac{2r_1 + 2r_2}{v}$, $T_E = T_{Duration} - T_0$
- $v = v_1 + v_2 = \sqrt{\frac{G\left(m_1 + m_2\right)}{a}}$, $\frac{dv}{dt} = 0$ at $0 \le t \le \frac{2r_2}{v}$ and $\frac{2r_1}{v} + T_E \le t \le T_{Duration}$

Conditions:

- $a \ge r_1 + r_2$ or $\left[P^2\left(m_1 + m_2\right)\right]^{1/3} \ge r_1 + r_2$
- $r_1 \ge r_2$

Assumptions:

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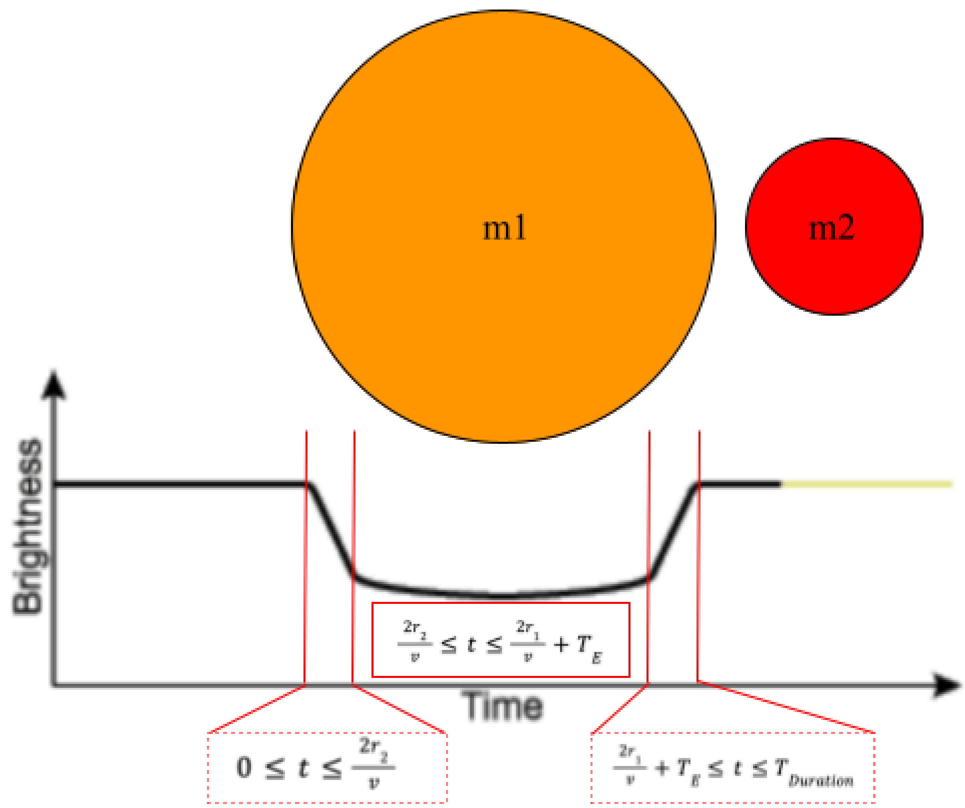

DESMOS SIMULATOR

Measure	Unit
Mass / $M_\odot$	$M_\odot$
Radius / $R_\odot$	$R_\odot$
Luminosity / $L_\odot$	$L_\odot$
Period / Days	Days
Projected Area / $R_\odot^{\ 2}$	$R_\odot^{\ 2}$
v	$R_\odot s^{-1}$

For m_2 eclipsing m_1

$$L_{Eclipse1}(t) = \left(1 - \frac{A_C(t)}{\pi r_1^{\ 2}}\right)L_1 + L_2, \qquad 0 \le t \le \frac{2r_2}{v}$$

$$L_{Eclipse2}(t) = \left(1 - \frac{r^2}{R^2}\right)L_1 + L_2, \qquad \frac{2r_2}{v} \le t \le \frac{2r_1}{v} + T_E$$

$$L_{Eclipse3}(t) = \left(1 - \frac{A_C\left(T_{Duration} - t\right)}{\pi r_1^{\ 2}}\right)L_1 + L_2, \qquad \frac{2r_1}{v} + T_E \le t \le T_{Duration}$$

$$A_C(t) = r_1^{\ 2} arccos\left(\frac{d(t)^2 + r_1^{\ 2} - r_2^{\ 2}}{2d(t)r_1}\right) + r_2^{\ 2} arccos\left(\frac{d(t)^2 + r_2^{\ 2} - r_1^{\ 2}}{2d(t)r_2}\right) - \frac{1}{2}\sqrt{\left(d(t)^2 - \left(r_2 - r_1\right)^2\right)\left(\left(r_1 + r_2\right)^2 - d(t)^2\right)}$$

$$v = \sqrt{\frac{G\left(m_1 + m_2\right)}{a}}$$

$$d(t) = \left(r_1 + r_2\right) - vt, \text{ where eclipse begins at t = 0}$$

$$T_{Duration} = \frac{P}{\pi} arcsin\left(\frac{r_1 + r_2}{a}\right)$$

Assuming:

1. No Limb Darkening (luminosity/projected area = const)
2. Orbital Inclination = 90 deg
3. Circular Orbit (e =0)
4. Perfectly Spherical Bodies (projected area being circles)

For m_2 eclipsing m_1

$$\begin{cases} \left(1 - \frac{A_C(t)}{\pi {r_1}^2}\right)L_1 + L_2, & 0 \leq t \leq \frac{2r_2}{v} \\ \left(1 - \frac{r^2}{R^2}\right)L_1 + L_2, & \frac{2r_2}{v} \leq t \leq \frac{2r_1}{v} + T_E \\ \left(1 - \frac{A_C\left(T_{Duration} - t\right)}{\pi {r_1}^2}\right)L_1 + L_2, & \frac{2r_1}{v} + T_E \leq t \leq T_{Duration} \end{cases}$$

Limitations:

- Attempts to use transit duration formula to solve at contact period
- Fails at minimum period/separation